classical value. Therefore, it is tentatively concluded that when tests are performed on isotropic elastic cylinders sufficiently free of imperfections, axially loaded in a rigid test machine, the classical value can be attained and a lower buckling load is not always inevitable.

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The Effect of a Cavity on Panel Vibration

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THE dynamic behavior of plates has recently acquired new interest in relation to their aeroelastic stability and excitation by noise. A prerequisite for studying such phenomena is the ability to predict the so-called natural modes and frequencies of the plate. One of the factors which may significantly alter the idealized model of a plate vibrating "in vacuo" is an underlying cavity. Here we present the results of an analysis of the system shown in Fig. 1. The model consists of a rectangular box of which one side is a vibrating plate.

We will assume the amplitude of motion is sufficiently small so that the linearized form of the governing flow equations and equations of elasticity may be employed. As is shown in numerous references, the flow field of compressible, inviscid fluid may be described by a velocity potential which satisfies the acoustic equation,

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2} \tag{1}$$

where φ is the velocity potential. The appropriate boundary conditions for our problem are

$$\frac{\partial\varphi}{\partial n} = 0 \quad \begin{cases} x = 0,l \\ y = 0,b \\ z = d \end{cases} \tag{2}$$

and

$$\partial\varphi/\partial z = \partial W/\partial t \qquad z = 0 \tag{3}$$

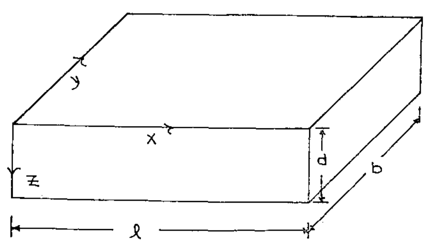

Fig. 1

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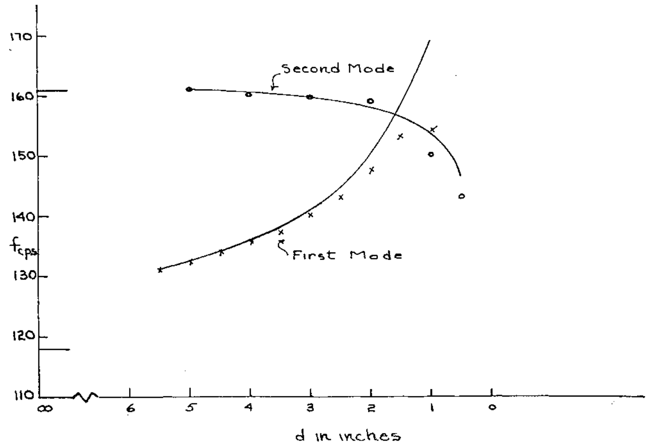

Fig. 2 Panel frequency vs cavity depth

where W is the plate deflection. We further assume a cosine-series expansion of the deflection,

$$W = \sum_m\sum_n\left[w_{mn}\cos\frac{m\pi x}{l}\cos\frac{n\pi y}{b}e^{i\omega t}\right] \tag{4}$$

Solving the system of Eqs. (1), (2), (3), using Eq. (4) gives

$$\varphi = i\omega e^{i\omega t}\sum_m\sum_n\left[\frac{w_{mn}}{v_{mn}}(\sinh v_{mn}z - \coth v_{mn}d\cosh v_{mn}z)\right]\cos\frac{m\pi x}{l}\cos\frac{n\pi y}{b} \tag{5}$$

where

$$v_{mn}^2 = \pi^2[(m/l)^2 + (n/b)^2] - (\omega/a)^2 \tag{6}$$

The pressure on the panel may be computed from the linearized form of Bernoulli's equation,

$$P = \rho\omega^2 e^{i\omega t}\sum_m\sum_n w_{mn}\frac{\coth v_{mn}d}{v_{mn}}\cos\frac{m\pi x}{l}\cos\frac{n\pi y}{b} \tag{7}$$

It may be noted that the nature of the pressure loading is dependent on the magnitude and sign of v_{mn}^2 and hence on the relative dimensions of panel and cavity. The quantity $a^2v_{mn}^2$ represents the difference in the squares of the frequencies of the natural cavity modes and the panel modes, and the pressure loading will be a maximum when this quantity is a minimum. However, for most panels, the frequency spectrum is such that all panel modes of interest lie between the cavity normal mode ($m = n = 0$) and the transverse mode ($m, n = 1, 0$). For practical panels and small cavity depths, it may be concluded that the principal effect on the

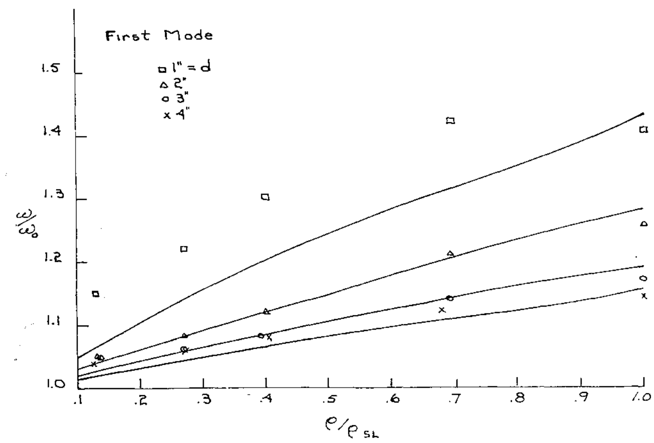

Fig. 3 Panel frequency vs cavity density

fundamental panel mode will be that of an aerodynamic spring, while for any antisymmetric panel mode, such as the second longitudinal mode, only aerodynamic mass effects will exist. The difference in effects is particularly significant in panel flutter where frequency differences are extremely important.

The equation of motion of the plate is

$$D \frac{\partial^4 W}{\partial x^4} + 2D \frac{\partial^4 W}{\partial x^2 \partial y^2} + D \frac{\partial^4 W}{\partial y^4} + m \frac{\partial^2 W}{\partial t^2} = P \qquad (8)$$

We shall study the particular case of a panel clamped on four edges. Thus the panel deflection will be represented by a series of the form

$$W = \sum_r \sum_s f_{rs} \psi_r(x) \psi_s(y) \qquad (9)$$

where

$$\psi_r = \cos \frac{(r-1)\pi x}{l} - \cos \frac{(r+1)\pi x}{l}$$

These primitive panel modes are not orthogonal; however, for the present purposes we may reasonably neglect the structural coupling and also the coupling between modes due to the cavity. Thus we shall treat each panel mode individually. Eqs. (7), (8), and (9) have been solved by the Galerkin method.

A comparison of the present theoretical results with previously obtained experimental results is presented in Figs. 2 and 3. In obtaining the theoretical curves the experimental values of the in vacuo frequencies were used. In Fig. 2 a plot of frequency vs cavity depth at standard atmospheric conditions is shown. Since considerably lower air densities are encountered in the wind tunnel, the variation of frequency with cavity density has been determined as well (see Fig. 3).

As may be seen, the agreement between theory and experiment is good except at the smaller cavity depths and densities. In particular, the change in the mode of minimum frequency predicted by theory is confirmed by the experimental data, with reasonable agreement on the cavity depth for which it occurs. One may hypothesize that an inclusion of nonlinear and/or viscous effects would bring the theory into better agreement with the experimental results at the smaller cavity depths and densities.

A theoretical calculation of the flutter dynamic pressure has been made for the tested panel with a cavity depth of three inches. The result indicates a decrease in flutter dynamic pressure of the order of 20 percent at Mach 2 with zero pressure differential across the panel.

In many practical cases only the fundamental mode will be modified by the aerodynamic-spring effect and the virtual-mass effect may be neglected. For this case a useful approximate formula may be written for the frequency of the fundamental mode—viz.,

$$K^2 \cong K^2_{\text{in vacuo}} + C\lambda \frac{l}{d}$$

where

$$K^2 = m\omega^2 l^4/D$$

$$\lambda = \rho a^2 l^3/D$$

and C is a constant determined by the boundary conditions. For a panel clamped on four sides $C = 0.44$, while for a simply-supported panel $C = 0.66$.

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Skin-Friction-Work Recovery by Aerodynamic Heating of Skin Coolants

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Nomenclature

c_p = specific heat
c_f = local skin-friction coefficient
c_h = local heat-transfer index (Stanton number)
g = acceleration due to gravity
J = mechanical equivalent of heat
ρ = local flow density
q = local heat-transfer rate
r = temperature-recovery factor
T = local temperature
V = local free-stream velocity
W = skin-friction work rate

Subscripts

aw = adiabatic wall
l = local
w = wall
t = total

THERMAL protection methods in current use on high-speed aircraft and missiles include heat sinks, ablation of coating materials, and transpiration cooling by fluids. To varying extents, the thermal energy transferred to vehicles using these methods of protection is lost. This note gives the result of an analysis that was made to determine the relationship of recoverable thermal energy to drag work when a skin coolant is used as an auxiliary means of propulsion or to implement the thrust of the main propulsive elements.

The rate of heat transfer per unit area on a flat plate can be written

$$q = c_h c_p \rho V g (T_{aw} - T_w) \qquad (1)$$

and the rate of skin-friction work in Btu's by

$$W = 1/2 c_f \rho V^3/J \qquad (2)$$

To determine the ratio of the heat recovered by the coolant to the skin-friction work, Eq. (1) is divided by Eq. (2):

$$q/W = 2c_h c_p g\, J(T_{aw} - T_w)/(c_f V^2) \qquad (3)$$

When the Reynolds analogy between the Stanton number and the skin-friction coefficient ($c_f = 2c_h$), as described by Truitt,[1] is used and the expression for velocity is taken from the energy equation, $V^2 = 2c_p Jg(T_t - T_l)$, Eq. (3) reduces to

$$\frac{q}{W} = \frac{1}{2} \frac{(T_{aw} - T_w)}{(T_t - T_l)} \qquad (4)$$

or, substituting the temperature-recovery factor, $r = (T_{aw} - T_l)/(T_t - T_l)$,

$$\frac{q}{W} = \frac{1}{2} r - \frac{1}{2} \frac{(T_w - T_l)}{(T_t - T_l)} \qquad (5)$$

The ratio of recoverable energy to the flow-friction work is shown in Fig. 1 for surface angles of 0°, 10°, and 20° at a free-stream Mach number of 10. While the lower wall temperature shows better efficiency, the jet propulsive efficiency of the coolant decreases as the jet temperature decreases. However, the coolant gains temperature as it flows within

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